



The University of Georgia

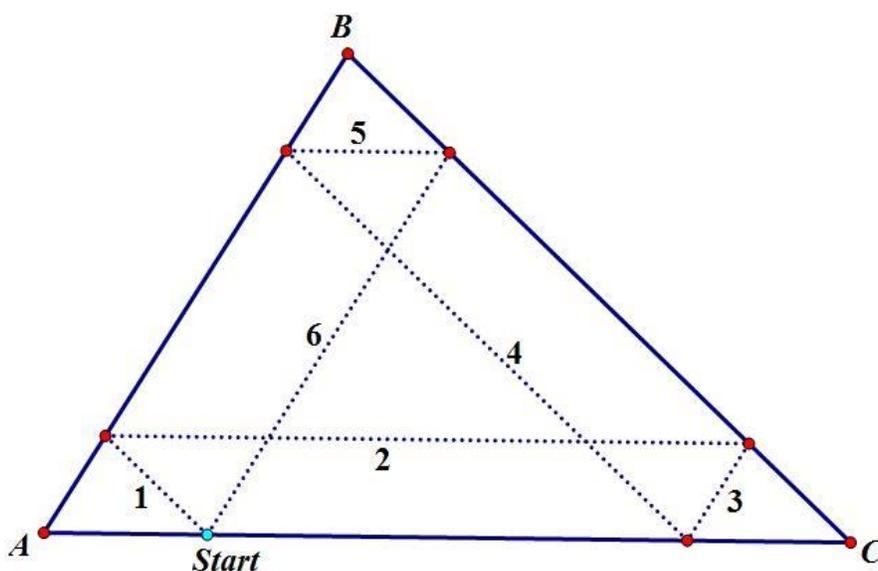
Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

Bouncing Barney By: Lindsey Harrison

Objective: In this exploration, we will investigate various questions concerning the Bouncing Barney scenario. Through this context, we will explore parallel transversals, ratios, and other triangle properties.

Prompt: Barney is in the triangular room shown below. He walks from a point on BC parallel to AC. When he reaches AB, he turns and walks parallel to BC. When he reaches AC, he turns and walks parallel to AB. ([GSP FILE](#))

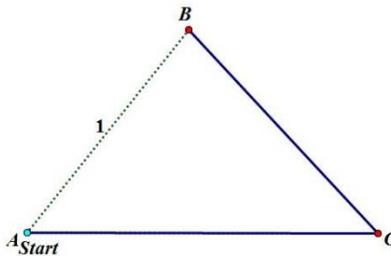
1. Prove that Barney will eventually return to his starting point.
2. How many times will Barney reach a wall before returning to his starting point?
3. Depending on where Barney starts walking, will he always travel the same distance?
4. Anything special about the path if he starts at the centroid or the orthocenter?



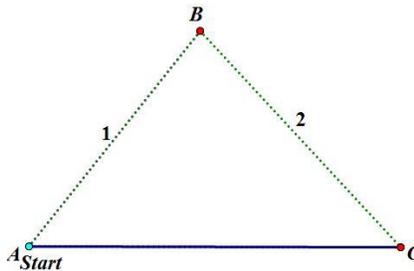
Will Barney always return to his starting point?

From the GSP animation, it appears that Barney will always return to his starting point, but this does not constitute a mathematical proof. There are two subsequent cases we need to explore.

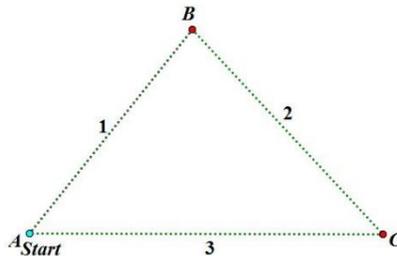
Case 1: First Barney starts at a corner of the room (an arbitrary vertex – A). There are three parallels since there are three sides of the triangular room. If Barney walks a path starting at vertex A parallel to length BC, then Barney will have to travel through walls outside of the room. Thus the two other paths he could take would be one parallel to AB or one parallel to AC. In each case Barney will be walking the length of a side of the triangular room. Again, without loss of generality, we will choose that Barney walks the path starting at vertex A parallel to AB. Since the segment AB is parallel to itself by the reflexive property, this path is valid.



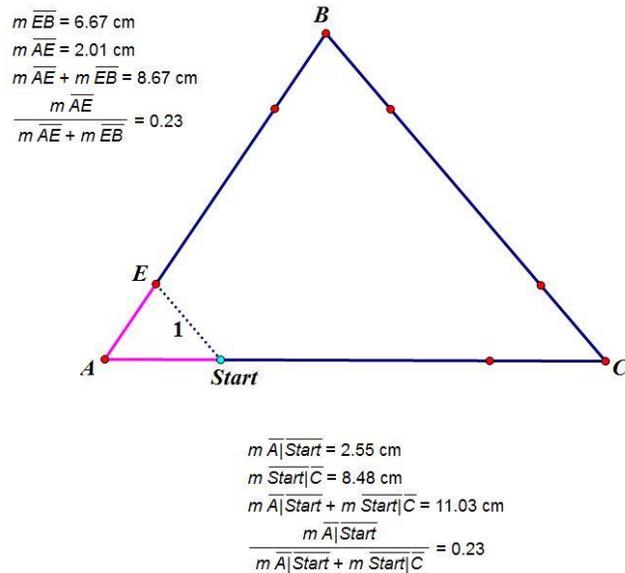
Barney's second location will then be at vertex B. The same argument above follows since the starting selection of vertex A was arbitrary. If he walked a path parallel to AC this would take him outside of the room. Thus, he only have paths parallel to AB or BC. He could travel a path parallel to AB in which he would turn around and walk the same path back to his starting point. He could also travel a path parallel to BC which would be a path along the side BC.



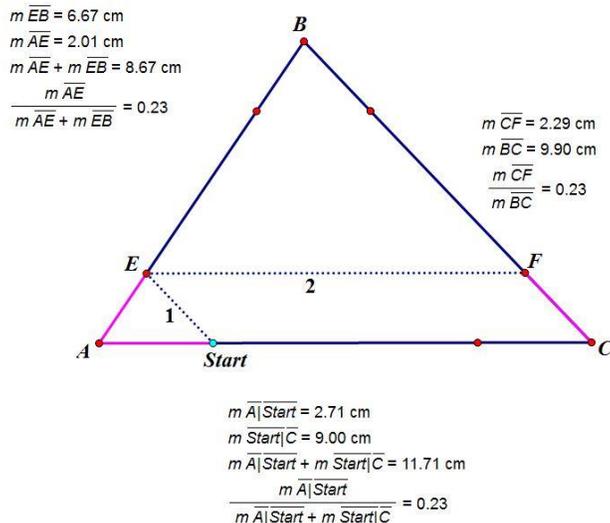
In a similar manner to the above two arguments, after Barney is at vertex C he would have to travel a path parallel to segment AC or BC. Traveling BC would have Barney retracing his steps and traveling AC would return Barney to his original starting point at vertex A. Since our selection of a starting vertex was arbitrary, the above proof holds for all cases when Barney starts his travels at a vertex.



Case 2: Barney starts at an arbitrary point along AC inside of the triangle. In this proof we will look at the distance from a vertex to the point where Barney hits the wall before he turns and begins walking again. The paths Barney takes are along parallel lines which have the property that they intersect triangle lets in a proportional manner. Look at the following example and explore the relationship in the following [GSP FILE](#).



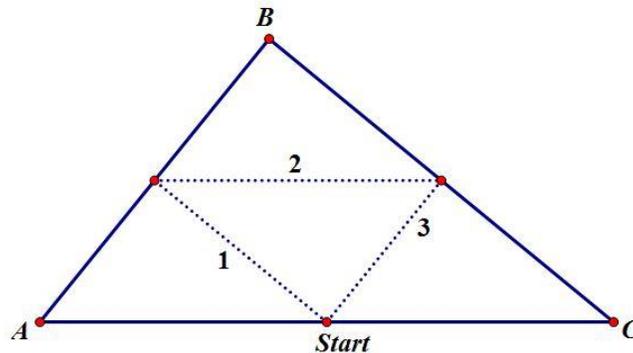
As we can see, the ratio of the part (shown in pink) to the whole triangle side is constant. This is because parallels always intersect line segments in a proportional relationship (it is for this reason that when we trisect a line we use parallel transversals).



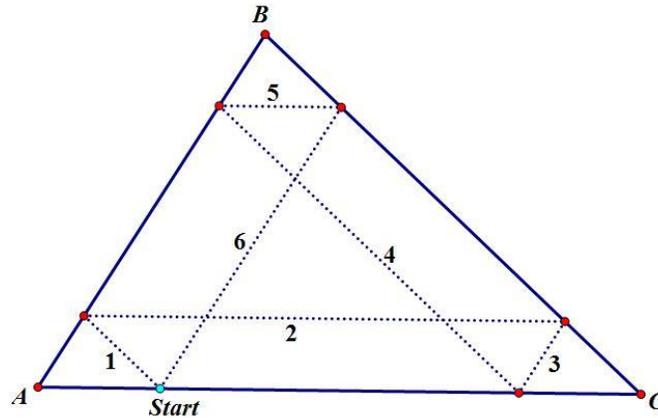
This property is maintained throughout Barney's journey and thus we can guarantee after his trip that Barney will end up at a point that is the same proportional distance we recorded in each step. The only unique point that satisfies this condition will be his original starting point.

How many times will Barney reach a wall before returning to his starting point?

If Barney starts at the midpoint of the line segment AC (as shown below) then he will reach two walls before returning to his starting point.

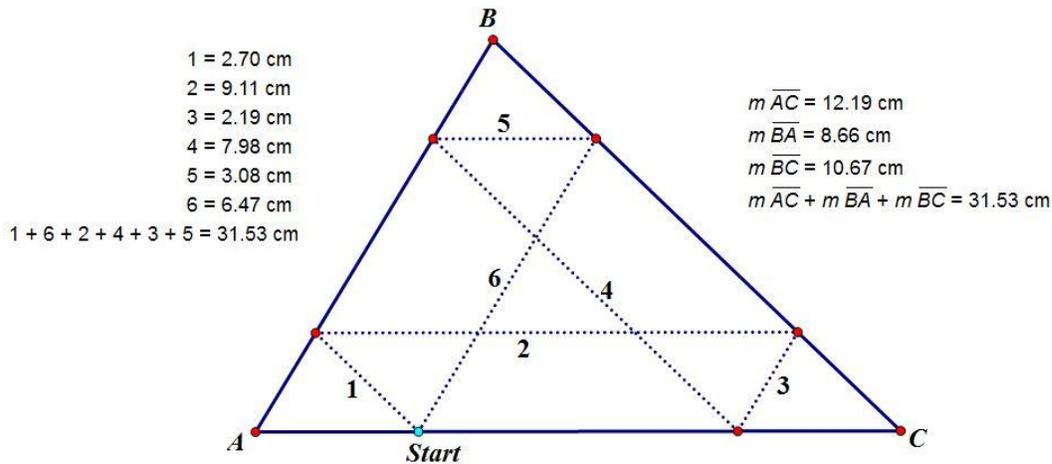


If Barney starts at a point along the line segment AC other than the midpoint he will reach five walls before returning to the starting point.

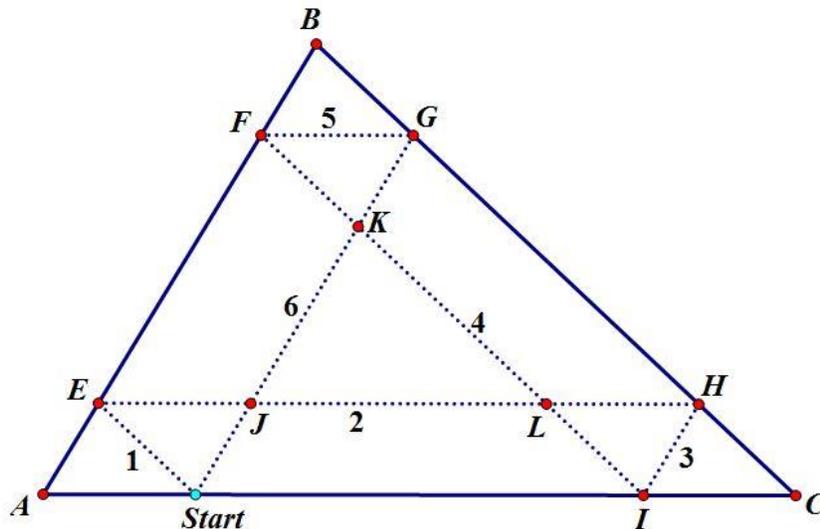


To generalize this conjecture, we could express the total number of walls Barney reaches before returning to his starting location as $n-1$ where n represents the total number of paths Barney took on his route. This is because with each progression in his path Barney is reaching a new wall; however, we must subtract away one because we do not count the wall after the final stage of his path because this is the wall where he originally began his trek.

Depending on where Barney starts walking, will he always travel the same distance?



Through an initial investigation, it appears that the distance Barney travels will always be the same distance as the perimeter of the triangle as long as he is not starting at the midpoint of a side (we will investigate that later). The first case when Barney starts at a vertex is obvious since he is traveling the lengths of the sides of the triangle. Since he travels all three sides before returning to this starting vertex, he has traversed a distance equal to that of the perimeter. Next we will explore the case when Barney starts at a point on side AC that is not the midpoint.



Our goal in this proof is that we want to show the following:

$$AB+BC+CA = SE+EJ+JL+LH+HI+IL+LK+KF+FG+GK+KJ+JS$$

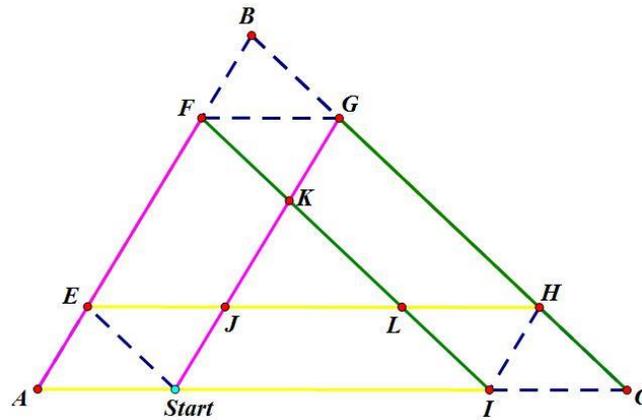
In plain terms we want to show that the distance Barney travels (right side of the equation) is equal to the distance around the perimeter of the triangle (left side of the equation). We can break down each side of the triangle as follows:

$$\begin{aligned}
 AB &= AE+EF+FB \\
 BC &= BG+GH+HC \\
 CA &= CI+IS+SA
 \end{aligned}$$

Using substitution we have the following that we want to prove:

$$AE+EF+FB+BG+GH+HC+CI+IS+SA= SE+EJ+JL+LH+HI+IL+LK+KF+FG+GK+KJ+JS$$

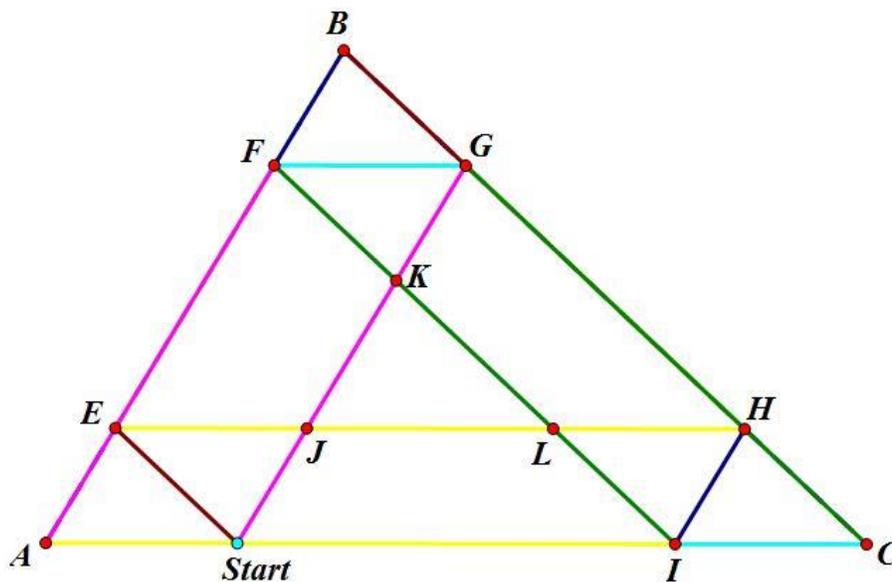
None of the above terms automatically cancel out since Barney is not walking any sides of the triangle in this scenario so let's get to work. First we can look at a set of parallel lines AF and SG. We know they are parallel by construction. They are also intersected by a pair of parallel transversals (FG and AS). This tells us that AF is congruent to SG. Therefore we have that $AE+EF=SJ+JK+KG$. With similar reasoning we have that $GH+HC=IL+LK+KF$. Also with that reasoning we have $AS+SI=EJ+JL+LH$. The diagram below shows the corresponding segments we have worked with in matching colors. We now only have to deal with the dotted line segments in blue that have not been addressed.



We know that SE is congruent to BG since they are parallel by construction and they are both intersected by a set of parallel transversals (AB and SG). With same reasoning we know that FG is congruent to IC and HI is congruent to FB. In conclusion we have shown that

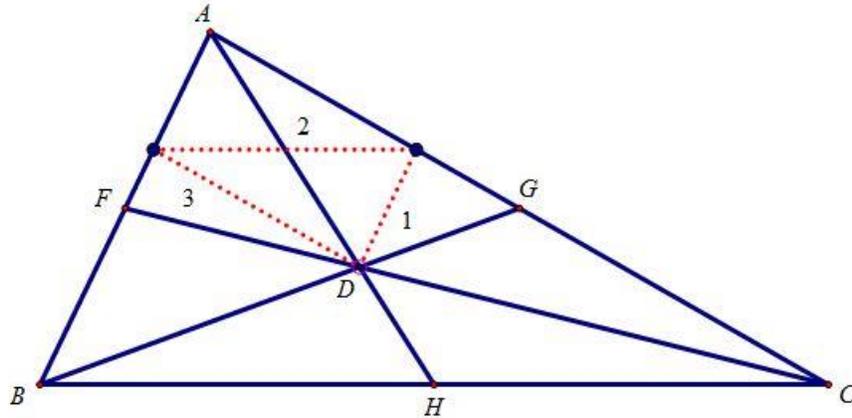
$$AE+EF+FB+BG+GH+HC+CI+IS+SA= SE+EJ+JL+LH+HI+IL+LK+KF+FG+GK+KJ+JS$$

The complete corresponding parts can be seen in the diagram below by matching colors.



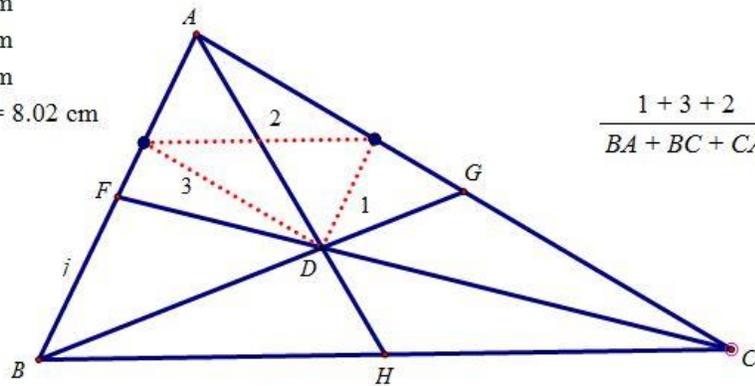
Anything special about the path if he starts at the centroid or the orthocenter?

First let us look at the path Barney takes if he starts at the centroid (point D).



The path Barney takes is outlined in red dotted lines. The path is similar to the midpoint path in that it only takes three walks to get back to the original starting point and that he only reaches two walls before his trip is over. Let us see if we can determine how long Barney walks when he takes a path starting at the centroid. We can see below that the distance Barney travels when starting at the centroid is always 1/3 of the perimeter of the triangle. This is the shortest path we have found thus far.

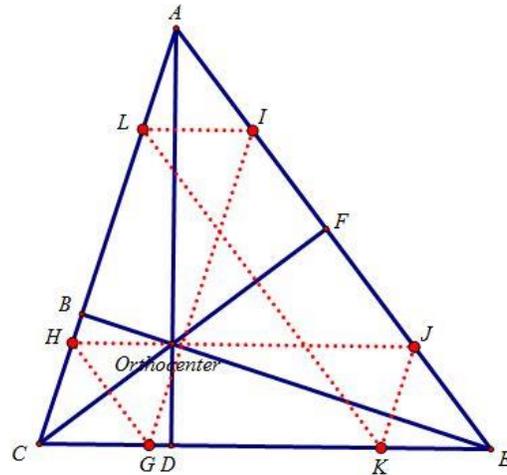
- 1 = 1.73 cm
- 2 = 3.32 cm
- 3 = 2.97 cm
- 1 + 3 + 2 = 8.02 cm



$$\frac{1 + 3 + 2}{BA + BC + CA} = 0.33$$

- BA = 5.20 cm
- BC = 9.95 cm
- CA = 8.92 cm
- BA + BC + CA = 24.07 cm

Next let us look at the path Barney takes if he starts at the orthocenter.



The path Barney takes is outlined in red dotted lines. The path is similar to the original path where Barney started at an arbitrary point (not the midpoint) on a triangle side. With this route however Barney reaches six walls instead of five since he is not returning at a wall as his starting point. Let us see if we can determine how long Barney walks when he takes a path starting at the orthocenter. We can see below that the distance Barney travels when starting at the orthocenter is always equal to the perimeter of the triangle.

$$m \overline{GH} = 1.99 \text{ cm}$$

$$m \overline{IG} = 5.16 \text{ cm}$$

$$m \overline{H|Orthocenter} = 1.56 \text{ cm}$$

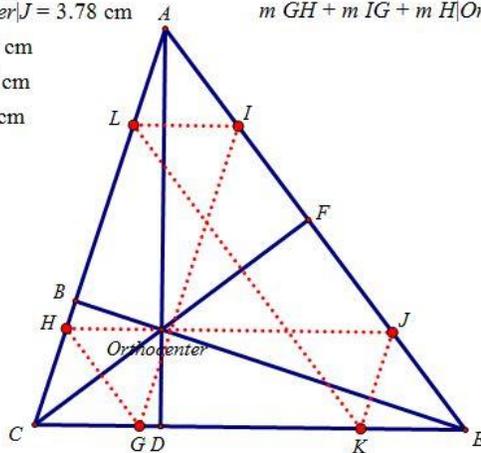
$$m \overline{Orthocenter|J} = 3.78 \text{ cm}$$

$$m \overline{KL} = 6.20 \text{ cm}$$

$$m \overline{JK} = 1.66 \text{ cm}$$

$$m \overline{LI} = 1.71 \text{ cm}$$

$$m \overline{GH} + m \overline{IG} + m \overline{H|Orthocenter} + m \overline{Orthocenter|J} + m \overline{KL} + m \overline{JK} + m \overline{LI} = 22.07 \text{ cm}$$



$$m \overline{AC} = 6.82 \text{ cm}$$

$$m \overline{CE} = 7.05 \text{ cm}$$

$$m \overline{EA} = 8.20 \text{ cm}$$

$$m \overline{AC} + m \overline{CE} + m \overline{EA} = 22.07 \text{ cm}$$